Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
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**Pearson Edexcel International Advanced Level**

Centre Number | Candidate Number

**Time** 1 hour 30 minutes

**Paper reference** **WFM01/01**

# Mathematics

## International Advanced Subsidiary/Advanced Level
## Further Pure Mathematics F1

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

*Turn over*


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**1.** (i) $$f(x) = x^3 + 4x - 6$$

(a) Show that the equation $f(x) = 0$ has a root $\alpha$ in the interval $[1, 1.5]$

**(2)**

(b) Taking 1.5 as a first approximation, apply the Newton Raphson process twice to $f(x)$ to obtain an approximate value of $\alpha$. Give your answer to 3 decimal places. Show your working clearly.

**(4)**

(ii) $$g(x) = 4x^2 + x - \tan x$$

where $x$ is measured in radians.

The equation $g(x) = 0$ has a single root $\beta$ in the interval $[1.4, 1.5]$

Use linear interpolation on the values at the end points of this interval to obtain an approximation to $\beta$. Give your answer to 3 decimal places.

**(4)**

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**Question 1 continued**

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**Question 1 continued**

**(Total 10 marks)**

**Q1**

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**2.** The complex numbers $z_1$, $z_2$ and $z_3$ are given by

$$z_1 = 2 - \mathrm{i} \qquad z_2 = p - \mathrm{i} \qquad z_3 = p + \mathrm{i}$$

where $p$ is a real number.

(a) Find $\dfrac{z_2 z_3}{z_1}$ in the form $a + b\mathrm{i}$ where $a$ and $b$ are real. Give your answer in its simplest form in terms of $p$.

**(3)**

Given that $\left|\dfrac{z_2 z_3}{z_1}\right| = 2\sqrt{5}$

(b) find the possible values of $p$.

**(4)**

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**Q2**

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**3.** The triangle $T$ has vertices $A(2, 1)$, $B(2, 3)$ and $C(0, 1)$.

The triangle $T'$ is the image of $T$ under the transformation represented by the matrix

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(a) Find the coordinates of the vertices of $T'$

**(2)**

(b) Describe fully the transformation represented by **P**

**(2)**

The $2 \times 2$ matrix **Q** represents a reflection in the $x$-axis and the $2 \times 2$ matrix **R** represents a rotation through 90° anticlockwise about the origin.

(c) Write down the matrix **Q** and the matrix **R**

**(2)**

(d) Find the matrix **RQ**

**(2)**

(e) Give a full geometrical description of the single transformation represented by the answer to part (d).

**(2)**

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**(Total 10 marks)**

Q3

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**4.** A rectangular hyperbola $H$ has equation $xy = 25$

The point $P\left(5t, \frac{5}{t}\right)$, $t \neq 0$, is a general point on $H$.

(a) Show that the equation of the tangent to $H$ at $P$ is $t^2y + x = 10t$

**(4)**

The distinct points $Q$ and $R$ lie on $H$. The tangent to $H$ at the point $Q$ and the tangent to $H$ at the point $R$ meet at the point $(15, -5)$.

(b) Find the coordinates of the points $Q$ and $R$.

**(4)**

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**Q4**

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**5.** $$\mathrm{f}(x) = (9x^2 + d)\left(x^2 - 8x + (10d + 1)\right)$$

where $d$ is a positive constant.

(a) Find the four roots of $\mathrm{f}(x)$ giving your answers in terms of $d$.

**(3)**

Given $d = 4$

(b) Express these four roots in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$.

**(2)**

(c) Show these four roots on a single Argand diagram.

**(2)**

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**(Total 7 marks)**

Q5

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**6.** The parabola $C$ has Cartesian equation $y^2 = 8x$

The point $P(2p^2, 4p)$ and the point $Q(2q^2, 4q)$, where $p, q \neq 0$, $p \neq q$, are points on $C$.

(a) Show that an equation of the normal to $C$ at $P$ is

$$y + px = 2p^3 + 4p$$

**(5)**

(b) Write down an equation of the normal to $C$ at $Q$

**(1)**

The normal to $C$ at $P$ and the normal to $C$ at $Q$ meet at the point $N$

(c) Show that $N$ has coordinates

$$\left(2(p^2 + pq + q^2 + 2), -2pq(p + q)\right)$$

**(5)**

The line $ON$, where $O$ is the origin, is perpendicular to the line $PQ$

(d) Find the value of $(p + q)^2 - 3pq$

**(5)**

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**7.** (a) Prove by induction that for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$$

**(5)**

(b) Hence show that

$$\sum_{r=1}^{n} (r^2 + 2) = \frac{n}{6}(an^2 + bn + c)$$

where $a$, $b$ and $c$ are integers to be found.

**(4)**

(c) Using your answers to part (b), find the value of

$$\sum_{r=10}^{25} (r^2 + 2)$$

**(2)**

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**Question 7 continued**

**(Total 11 marks)**

Q7

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**8.** Prove by induction that $4^{n+2} + 5^{2n+1}$ is divisible by 21 for all positive integers $n$.

**(6)**

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Q8

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**TOTAL FOR PAPER: 75 MARKS**

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